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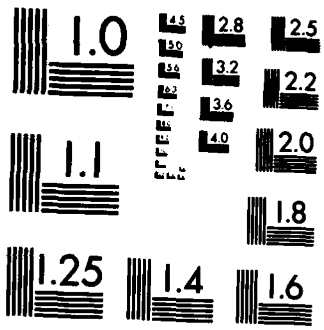
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A Propagator Expansion Method for Solving Linearized Plasma Kinetic Equations With Collisions

JOHN R. JASPERSE

25 June 1984

IONOSPHERIC PHYSICS DIVISION PROJECT 2310
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In this paper, we present a propagator expansion method for solving linearized plasma kinetic equations with collisions. The essence of the method is the representation and use of the collisional propagator for a given problem as an expansion in powers of the collision frequency. For the linearized Balescu-Lenard collision operator and for the zero-order distribution function Maxwellian, we obtain an exact expression for the collisional damping rate in the long wavelength approximation to the first order in the plasma parameter.

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Preface

I should like to acknowledge important discussions with B. Basu, T.S. Chang, and J.M. Retterer. This work was sponsored by the Air Force Office of Scientific Research.

Contents

A Propagator Expansion Method For Solving Linearized Plasma Kinetic Equations With Collisions

1. INTRODUCTION

In 1946, Landau[1] gave a method for solving linearized plasma kinetic problems where discrete particle interactions were neglected. Subsequent studies[2-4] of collisional plasmas have employed methods of solution tailored to a specific form for the collision operator. In this paper, we give a general expansion method for solving linearized plasma kinetic problems when collisions are included. The method can be applied to a wide class of collision operators and has, for example, produced closed-form results for the collisional dielectric function for the Balescu[5]-Lenard[6] collision operator.

The essence of the idea presented in this paper is the derivation of a collisional propagator in terms of the collisionless propagator for the Vlasov equation and the linearized collision operator, and the representation and use

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1. Landau, L.D. (1946) Zh. Eksp. Teor. Fiz. 16:574 and J. Phys. (USSR) 10:25.
2. Bhatnagar, P.L., Gross, E.P., and Krook, M. (1954) Phys. Rev. 94:511.
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of the collisional propagator either as a series solution in powers of the collision frequency ν or as an asymptotic expansion as the collision frequency tends to zero.

2. KINETIC EQUATIONS

The kinetic equations in the electrostatic approximation for a collisional plasma are:

$$\left\{\frac{\partial}{\partial t} + \vec{v}\cdot\frac{\partial}{\partial \vec{r}} + \left[\frac{q_e}{m_e}\,\vec{E}(\vec{r},t) + \vec{a}_0(\vec{v})\right]\cdot\frac{\partial}{\partial \vec{v}}\right\} f(\vec{r},\vec{v},t) = \mathcal{L}(f) \ , \tag{1}$$

$$\frac{\partial}{\partial \vec{r}}\cdot \vec{E}(\vec{r},t) = 4\pi\left[n_i q_i + q_e \int d^3v\, f(\vec{r},\vec{v},t)\right] , \tag{2}$$

where $\vec{E}$ is the self-consistent, electrostatic field; $\vec{a}_0$ is the force-per-unit mass due to spatially uniform, stationary fields; and $\mathcal{L}(f)$ is the nonlinear collision operator that acts on the velocity-space coordinates of the electron distribution function f. The ions are assumed to provide a fixed neutralizing background with charge density $n_i q_i$ while the other quantities have their usual meanings. We seek a perturbation solution of the form

$$f(\vec{r},\vec{v},t) = f_0(\vec{v}) + f_1(\vec{r},\vec{v},t) \ , \tag{3}$$

$$\vec{E}(\vec{r},t) = \vec{E}_1(\vec{r},t) \ , \tag{4}$$

where f_0 is the zero-order solution and f_1 and $\vec{E}_1$ are perturbed quantities. The first-order equations using abbreviated notation are:

$$\left(\frac{\partial}{\partial t} + \vec{v}\cdot\frac{\partial}{\partial \vec{r}} + \vec{a}_0\cdot\frac{\partial}{\partial \vec{v}} - L^{\nu}\right) f^{\nu}(t) = S^{\nu}(t) \ , \tag{5}$$

$$\frac{\partial}{\partial \vec{r}}\cdot\frac{\partial}{\partial \vec{r}}\,\phi^{\nu}(t) = -\,4\pi q_e \int d^3v\, f^{\nu}(t) \ , \tag{6}$$

$$S^{\nu}(t) = \frac{q_e}{m_e}\,\frac{\partial}{\partial \vec{r}}\,\phi^{\nu}(t)\cdot\frac{\partial}{\partial \vec{v}}\, f_0 \ , \tag{7}$$

where the subscript 1 on the perturbed quantities has been deleted and the superscript ν is used to denote collisional dependence, $\phi^{\nu}(t)$ is the electrostatic potential, and L^{ν} denotes the linearized collision operator. The operator equation for the collisional propagator operator $U^{\nu}(t-t')$, is

$$\left(\frac{\partial}{\partial t} + \vec{v} \cdot \frac{\partial}{\partial \vec{r}} + \vec{a}_0 \cdot \frac{\partial}{\partial \vec{v}} - L^{\nu}\right) U^{\nu}(t-t') = 0 \quad , \tag{8}$$

where $U^{\nu}(t-t')$ is given by

$$U^{\nu}(t-t') = \int d^3r' \int d^3v' \, G^{\nu}(\vec{r}-\vec{r}', \vec{v}, \vec{v}', t-t') \quad , \tag{9}$$

and where $G^{\nu}(\vec{r}-\vec{r}', \vec{v}, \vec{v}', t-t')$, the Green's function for the problem, is translationally invariant in space and time. Here, $U^{\nu}(\tau)$ is subject to the causality condition that $U^{\nu}(\tau)=0$ for $\tau < 0$ and the initial condition that $U^{\nu}(0)=1$. In terms of $U^{\nu}(t-t')$ the solution for $f^{\nu}(t)$ is

$$f^{\nu}(t) = U^{\nu}(t) \, f^{\nu}(0) + \int_0^t dt' \, U^{\nu}(t-t') \, S^{\nu}(t') \quad , \tag{10}$$

where $f^{\nu}(0)$ denotes the initial value of $f^{\nu}(t)$. It is convenient to transform Eqs. (6) through (10) to $\vec{k}$-$\vec{v}$-ω space using

$$f^{\nu}(t) = T_{\vec{k},\omega}^{-1} \, \tilde{f}^{\nu}(\vec{k}, \vec{v}, \omega) = T_{\vec{k},\omega}^{-1} \, \tilde{f}^{\nu}(\omega) \quad , \tag{11}$$

$$\tilde{f}^{\nu}(\omega) = T_{\vec{r},t} \, f^{\nu}(\vec{r}, \vec{v}, t) = T_{\vec{r},t} \, f^{\nu}(t) \quad , \tag{12}$$

$$T_{\vec{k},\omega}^{-1} = \int \frac{d^3k}{(2\pi)^3} \int_C \frac{d\omega}{2\pi} \exp(i\vec{k} \cdot \vec{r} - i\omega t) \quad , \tag{13}$$

$$T_{\vec{r},t} = \int d^3r \int_0^{\infty} dt \, \exp(-i\vec{k} \cdot \vec{r} + i\omega t) \quad , \tag{14}$$

where $\vec{k}$ is real, Im $\omega \geqq b > 0$, and the contour C runs from $-\infty + ib$ to $+\infty + ib$. Here b is chosen sufficiently large enough to ensure that the t-integration converges.[7] The equations in $\vec{k}$-$\vec{v}$-ω space are:

$$\tilde{f}^{\nu}(\omega) = \tilde{U}^{\nu}(\omega) \left[f^{\nu}_{\vec{k}}(0) + i \frac{q_e}{m_e} \cdot \tilde{\phi}^{\nu}(\omega) \, \vec{k} \cdot \frac{\partial}{\partial \vec{v}} f_0 \right] , \tag{15}$$

7. Titchmarsh, E. C. (1948) Introduction to the Theory of Fourier Integrals, Oxford University Press, London, Chapter 1.

$$\tilde{\phi}^{\nu}(\omega) = \frac{4\pi q_e}{k^2}\left[\frac{1}{\varepsilon^{\nu}(\omega)}\right]\int d^3v\, \tilde{U}^{\nu}(\omega)\, f^{\nu}_{\vec{k}}(0) \quad , \tag{16}$$

$$\varepsilon^{\nu}(\omega) = 1 - i\left(\frac{4\pi q_e^2}{m_e k^2}\right)\int d^3v\, \tilde{U}^{\nu}(\omega)\, \vec{k}\cdot\frac{\partial}{\partial\vec{v}} f_0 \quad , \tag{17}$$

where $f^{\nu}_{\vec{k}}(0)$ is the spatial transform of $f^{\nu}(0)$. The operator $\tilde{U}^{\nu}(\omega)$ is

$$\tilde{U}^{\nu}(\omega) = \int d^3v'\, \tilde{G}^{\nu}(\vec{k},\vec{v},\vec{v}',\omega) \quad , \tag{18}$$

and the Green's function in $\vec{k}$-$\vec{v}$-ω space is

$$\tilde{G}^{\nu}(\vec{k},\vec{v},\vec{v}',\omega) = T_{\vec{R},\tau}\; G^{\nu}(\vec{R},\vec{v},\vec{v}',\tau) \quad . \tag{19}$$

3. EQUATION FOR THE COLLISIONAL PROPAGATOR

In order to derive an equation for $\tilde{U}^{\nu}(\omega)$, we consider a collisionless propagator operator, $U(t-t')$, which obeys the operator equation

$$\left(\frac{\partial}{\partial t} + \vec{v}\cdot\frac{\partial}{\partial\vec{r}} + \vec{a}_0\cdot\frac{\partial}{\partial\vec{v}}\right)U(t-t') = 0 \tag{20}$$

and is subject to the same causality and initial conditions as those imposed on $U^{\nu}(\tau)$. We return to Eq. (8) and write an operator equation for $U^{\nu}(t-t')$ in terms of $U(t-t')$ and L^{ν}:

$$U^{\nu}(t-t') = U(t-t') + \int_{t'}^{t} dt''\,U(t-t'')\, L^{\nu}\, U^{\nu}(t''-t') \quad , \tag{21}$$

where we have used causality and the initial conditions. Transforming Eq. (21) to $\vec{k}$-$\vec{v}$-ω space, we obtain

$$\tilde{U}^{\nu}(\omega) = \tilde{U}(\omega) + \tilde{U}(\omega)\, L^{\nu}\, \tilde{U}^{\nu}(\omega) \quad . \tag{22}$$

Solving Eq. (22) formally, we obtain

$$\tilde{U}^{\nu}(\omega) = [\, 1 - \tilde{U}(\omega)\, L^{\nu}\,]^{-1}\, \tilde{U}(\omega) \quad . \tag{23}$$

Equations (15) through (17) and Eq. (23) give a formal solution to the collisional, initial-value problem.

4. ASYMPTOTIC EXPANSIONS AS $\nu \to 0$

In order to pursue a solution by the expansion method, we introduce a collision frequency ν and write $L^{\nu} = \nu L$. In most cases, ν will be small compared to other frequencies of interest. For a wide class of collision operators including integral, differential, and integral-differential, and for some continuity and integrability conditions on the functions that appear in the problem, it can be shown that the iteration of Eq. (22) yields an asymptotic expansion for $\widetilde{U}^{\nu}(\omega)$ as $\nu \to 0$:

$$\widetilde{U}^{\nu}(\omega) \sim \widetilde{U}(\omega) + \nu\, \widetilde{U}(\omega)\, L\, \widetilde{U}(\omega) + \dots + \nu^{n}[\widetilde{U}(\omega)\, L]^{n}\, \widetilde{U}(\omega) + \dots \quad . \tag{24}$$

By this we mean that if $\widetilde{g}(\omega) = \widetilde{g}(\vec{k}, \vec{v}, \omega)$ and $\widetilde{f}^{\nu}(\omega) = \widetilde{U}^{\nu}(\omega)\, \widetilde{g}(\omega)$, then the action of the iterated $\widetilde{U}^{\nu}(\omega)$, given by Eq. (24), on $\widetilde{g}(\omega)$ generates an asymptotic expansion for $\widetilde{f}^{\nu}(\omega)$ as $\nu \to 0$. Similar results can be obtained for the iterated $\varepsilon^{\nu}(\omega)$. If the iterated $\widetilde{f}^{\nu}(\omega)$ is a divergent asymptotic expansion, then Eq. (24) may not be truncated and used to approximate the initial-value problem as $t \to \infty$ because time secularities will appear in the solution when transformed to $\vec{r}$-$\vec{v}$-t space. However, even if the iterated $\varepsilon^{\nu}(\omega)$ is also divergent but $\nu/\omega_p \ll 1$, where ω_p is the plasma frequency, then $\varepsilon^{\nu}(\omega)$ may be truncated at first order in ν and set equal to zero to obtain an approximate solution for the roots of the dispersion relation. This provides a solution for the normal mode and stability problems accurate to order ν/ω_p. The expression for the collisional dielectric function truncated to the first order in ν, denoted by $\varepsilon^{(1)}(\vec{k}, \omega)$, is

$$\varepsilon^{(1)}(\vec{k}, \omega) = 1 - i\, \frac{\omega_p^2}{k^2}\, \{T_0(\vec{k}, \omega) + \nu\, T_1(\vec{k}, \omega)\} \quad , \tag{25}$$

$$T_n(\vec{k}, \omega) = \int d^3v\, [\widetilde{U}(\omega)\, L]^{n}\, \widetilde{U}(\omega)\, \vec{k} \cdot \frac{\partial}{\partial \vec{v}} f_0(\vec{v}) \quad , \tag{26}$$

where f_0 has been normalized to one.

5. UNIFORMLY CONVERGENT EXPANSIONS IN POWERS OF ν

For the special case when $\widetilde{U}(\omega)$ is the free-particle, collisionless propagator and L is an integral operator, Eq. (22) is an integral equation for $\widetilde{U}^{\nu}(\omega)$. In our study of problems of this type, we have found that if the functions that appear in the problem are continuous, integrable, and obey some reasonable uniform bounds, then the iteration procedure yields the Neumann series for the associated integral equation and

$$\widetilde{U}^{\nu}(\omega) = \widetilde{U}(\omega) + \sum_{n=1}^{\infty} \nu^n [\widetilde{U}(\omega)L]^n \, \widetilde{U}(\omega) \quad , \tag{27}$$

is the power-series solution for $\widetilde{U}^{\nu}(\omega)$, uniformly convergent in the neighborhood of ν=0. In this case the iterated $\varepsilon^{\nu}(\omega)$ is:

$$\varepsilon^{\nu}(\vec{k}, \omega) = 1 - i \frac{\omega_p^2}{k^2} \sum_{n=0}^{\infty} \nu^n T_n(\vec{k}, \omega) \quad , \tag{28}$$

$$T_n(\vec{k},\omega) = \int d^3v \left[\left(\frac{i}{\omega - \vec{k} \cdot \vec{v}} \right) L \right]^n \left(\frac{i}{\omega - \vec{k} \cdot \vec{v}} \right)$$
$$\times \vec{k} \cdot \frac{\partial}{\partial \vec{v}} f_0(\vec{v}) \quad , \tag{29}$$

where f_0 has been normalized to one and the expression for the free-particle, collisionless propagator has been used:

$$\widetilde{U}(\omega) = \int d^3v' \, \frac{i}{\omega - \vec{k} \cdot \vec{v}} \, \delta\,(\vec{v} - \vec{v}\,') \quad . \tag{30}$$

For problems of this type, it also follows that Eq. (28) is the power-series solution for $\varepsilon^{\nu}(\omega)$, uniformly convergent in the neighborhood of ν=0. If Eq. (27) can be summed in closed form and analytically continued for Im ω<b, then the initial-value problem is solved.

6. SYMMETRY PROPERTIES

It can be shown that $\tilde{G}^{\nu}(\vec{k},\vec{v},\vec{v}',\omega)^* = \tilde{G}^{\nu}(-\vec{k},\vec{v},\vec{v}',-\omega^*)$ with similar formulae for the other $\vec{k}$-$\vec{v}$-ω quantities discussed in Sections 2 and 3, and that the roots of the collisional dispersion relation, $\varepsilon^{\nu}(\omega)=0$, obey the relation $\omega^{\nu}(\vec{k}) = -\omega^{\nu}(-\vec{k})^*$. The same symmetry properties hold for each of the iterated $\vec{k}$-$\vec{v}$-ω quantities discussed in Sections 4 and 5 when truncated to the Nth order in ν.

7. SOME APPLICATIONS

For the free-particle, collisionless propagator, three types of linearized collision operators have been studied: integral, differential, and integral-differential.

As an example of the integral-type collision operator, we studied the simple BGK[2] collision operator, given by

$$L^{\nu} = \nu\,[-1 + f_0(\vec{v})\int d^3v'\,] \;, \tag{31}$$

where f_0 has been normalized to one. We found that the iterated $\tilde{U}^{\nu}(\omega)$ and the iterated $\varepsilon^{\nu}(\omega)$ are uniformly convergent power-series solutions in ν that agree with expansions of the exact results.

The linear simple Fokker-Planck collision operator is a differential operator and has been studied by Lenard and Bernstein.[3] It is

$$L^{\nu} = \nu\,\frac{\partial}{\partial\vec{v}}\cdot\left(\vec{v} + v_T^2\,\frac{\partial}{\partial\vec{v}}\right) \;, \tag{32}$$

where v_T is the electron thermal speed. *We found that the iterated* $\tilde{U}^{\nu}(\omega)$ *is a* divergent, asymptotic expansion as $\nu \to 0$ in the sense described above, yet the *iterated* $\varepsilon^{\nu}(\omega)$ *for* f_0 *Maxwellian is a uniformly convergent power-series solution* in ν and agrees with the expansion of the closed-form result for $\varepsilon^{\nu}(\omega)$. For *example, the solution to the dispersion relation in the long wavelength limit to* order ν yields the total damping rate $\Gamma^{\nu}_{\vec{k}}$, which is the sum of the Landau (collisionless) part $\gamma^{L}_{\vec{k}}$ and the collisional part $\gamma^{\nu}_{\vec{k}}$, where

$$\gamma^{\nu}_{\vec{k}} = -\,\nu[\;1/2 + 2(k/k_D)^2 + \ldots\,] \;. \tag{33}$$

Here k_D is the Debye wavenumber. Note that these results are obtained without solving a differential equation involving L^{ν}.

The linearized Balescu[5] - Lenard[6] collision operator is an integral-differential operator. We have found that the iterated $\tilde{U}^{\nu}(\omega)$ is a divergent, asymptotic expansion as $\nu \to 0$ and we have obtained closed-form results for the iterated $\varepsilon^{\nu}(\omega)$ to the first order in ν. Using these results, we have found the solution for the collisional dispersion relation in the long wavelength limit for the linearized Balescu - Lenard collision operator and for f_0 Maxwellian. We obtain the total damping rate $\Gamma^{\nu}_{\vec{k}}$ which is the sum of the Landau (collisionless) part $\gamma^{L}_{\vec{k}}$ and the collisional part $\gamma^{\nu}_{\vec{k}}$, where

$$\gamma^{\nu}_{\vec{k}} = -\frac{4}{5}\nu_{ee}\left(\frac{k}{k_D}\right)^2\left[1+\frac{69}{14}\left(\frac{k}{k_D}\right)^2+\ldots\right]$$

$$-\frac{1}{2}\nu_{ei}\left[1-2\left(\frac{k}{k_D}\right)^2+\ldots\right], \tag{34}$$

$$\nu_{ee} = \frac{4}{3}\pi^{1/2} n_0 \frac{q_e^4}{m_e^{1/2}T_e^{3/2}} \ell n\,\Lambda \ . \tag{35}$$

Here $\nu_{ei} = 2^{1/2}\nu_{ee}$, $k = |\vec{k}|$, k_D is the Debye wave number, n_0 is the unperturbed density, T_e is the unperturbed temperature, and $\ell n\,\Lambda$ is the Coulomb logarithm. Equation (34) is new and was obtained in collaboration with B. Basu. A detailed discussion of its derivation is quite lengthy and will be published elsewhere.

8. DISCUSSION

Equations (21) through (23) are the main results of this paper. They give equations for the collisional propagator in terms of the collisionless propagator and the linearized collision operator. An iterative solution of Eq. (22) or (23), given by Eq. (24), which is what we call the expansion method of solution, when substituted into Eqs. (15) through (17) provides a solution of the linearized plasma kinetic equations for a given collision operator. This method of solution as applied to the collisional plasma problem discussed here is new and is particularly useful when a direct solution of the problem cannot be found. For example, the problem of finding the direct solution of the linearized plasma kinetic equations with the Balescu-Lenard collision term and then using it to find the collisional damping of the electrostatic plasma waves appears to be intractable. But by applying the expansion method, we have been able to obtain the collisional damping rate for f_0 Maxwellian to the first order in the plasma parameter, which is given by Eqs. (34) and (35).

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2. Bhatnagar, P.L., Gross, E.P., and Krook, M. (1954) Phys. Rev. 94:511.
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